

## Complex number problem

<http://www.starmind.com/question/1182/>

**Abstract: A simple algebraic derivation of the answer, starting from a geometric consideration of the complex plane.**

First, see that  $e^{ix} = i$ , where  $x = \pi/2 + 2n\pi$  for each integer  $n$ , as multiplication (of 1) by  $\pi/2$  is a quarter-turn anti-clockwise around the conventional geometric map of the complex plane, and then any other rotation of  $2n\pi$  will give the same value.

Then raising both sides to the power  $i$ , we get

$e^{iix} = i^i$ , (concretely  $e^{i(\pi/2 + 2n\pi)} = i^i$ ) and  $e^{-x} = i^i$ .

So  $i^i = e^{-x}$ , where  $x = \pi/2 + 2n\pi$ , for each integer  $n$ . The expression is multivalued, and real for each value that it takes (as it involves only real constants). Taking  $n=0$  gives 0.207879576.... as the principle value.

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